| Ref # | Hits | Search Query | DBs | Default Operator | Plurals | Time Stamp |
|---|---|---|---|---|---|---|
| L1 | 2 | encrypt$4.ab. and optical.ab. and decrypt$5.ab. and algebraic.ab. | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 17:32 |
| L2 | 3 | ("6125183").URPN. | USPAT | OR | ON | 2005/06/11 17:32 |
| L3 | 2 | encrypt$4.ab. same optical.ab. same decrypt$5.ab. same algebraic.ab. | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 17:32 |
| L4 | 2 | encrypt$4 same optical same decrypt$5 same algebraic | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 17:33 |
| L5 | 42 | encrypt$4 same decrypt$5 same algebraic | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 17:33 |

| Ref # | Hits | Search Query | DBs | Default Operator | Plurals | Time Stamp |
|---|---|---|---|---|---|---|
| L1 | 1840 | encrypt$5.ab. and decrypt$5.ab. and (image or video or television or movie or film$5).ab. | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 15:45 |
| L2 | 1101317 | (image or video or television or movie or film$5) with (break$5 or divid$5 or partition$5 or part$5 or divis$5 or segment$5 or block$5 or piec$5) | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 15:46 |
| L3 | 3849 | encrypt$5 with (algebr$5 or discreet$5 or cyclotom$5 or circul$5 or ellipt$5 or polynom$5 or integer or power$5 or mathmat$5 or formul$5) | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 15:47 |
| L4 | 66 | L1 and L2 and L3 | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 15:48 |
| L5 | 34 | L4 and @ad<"20010330" | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 15:48 |
| L6 | 1 | ("6570990").URPN. | USPAT | OR | ON | 2005/06/11 15:53 |
| L7 | 5 | ("6240183").URPN. | USPAT | OR | ON | 2005/06/11 15:56 |
| L8 | 3 | ("6064738").URPN. | USPAT | OR | ON | 2005/06/11 15:59 |
| L9 | 0 | ("5974144").URPN. | USPAT | OR | ON | 2005/06/11 16:01 |
| L10 | 2 | ("5933499").URPN. | USPAT | OR | ON | 2005/06/11 16:02 |
| S1 | 2 | ("6505299").PN. | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | OFF | 2005/06/11 15:44 |
| S2 | 2 | ("6505299").URPN. | USPAT | OR | ON | 2005/06/10 14:46 |
| S3 | 70857 | encrypt$5 | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/10 15:17 |
| S4 | 1212028 | digital | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/10 15:17 |

| | | | | | | |
|---|---|---|---|---|---|---|
| S6 | 3210433 | image or picture or video or movie | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/10 15:18 |
| S7 | 8424069 | polynomial or "one or more" or power or integral or two | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/10 15:18 |
| S8 | 2368095 | cyclotom$4 or circul$4 or ellipt$6 or curv$5 | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/10 15:19 |
| S9 | 31738 | decrypt$5 | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/10 15:19 |
| S10 | 1 | S3 with S4 with S6 with S7 with S8 | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/10 15:21 |
| S11 | 9 | S3 same S4 same S6 same S7 same S8 | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 12:51 |
| S12 | 30 | cyclotom$4 with polynom$5 | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 13:34 |
| S13 | 3210433 | image or picture or video or movie | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 13:14 |
| S14 | 7267 | encrypt$5 with S13 | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 13:14 |
| S15 | 5 | S14 and S12 | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 13:15 |

| | | | | | | |
|---|---|---|---|---|---|---|
| S16 | 100 | S14 and cinema | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 13:16 |
| S17 | 83 | S16 and encrypt$5 and decrypt$5 | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 13:16 |
| S18 | 26 | S17 and @ad<"20000331" | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 13:16 |
| S19 | 5 | ("6141530").URPN. | USPAT | OR | ON | 2005/06/11 13:22 |
| S20 | 3210433 | image or picture or video or movie | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 13:42 |
| S21 | 2776 | (encrypt$5 with S20) and (decrypt$5 with S20) | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 13:44 |
| S22 | 66 | S21 and ((mathmat$6 or algebr$5 or formul$5 or unity or discreet) with (encrypt$5 or algorith$5)) | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 14:39 |
| S23 | 52 | ("5862218").URPN. | USPAT | OR | ON | 2005/06/11 13:53 |
| S24 | 2 | ("5933499").URPN. | USPAT | OR | ON | 2005/06/11 13:58 |
| S25 | 1 | ("6456985").URPN. | USPAT | OR | ON | 2005/06/11 13:58 |
| S26 | 1 | ("6363210").URPN. | USPAT | OR | ON | 2005/06/11 14:00 |
| S27 | 1 | ("6490352").URPN. | USPAT | OR | ON | 2005/06/11 14:34 |
| S28 | 1684 | S20.ab. and encrypt$5.ab. and decrypt$5.ab. | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 14:35 |
| S29 | 1692 | (S20.ab. or imaging.ab.) and encrypt$5.ab. and decrypt$5.ab. | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 14:36 |

| | | | | | | |
|---|---|---|---|---|---|---|
| S30 | 395590 | S20 with (partition$5 or divid$5 or separat$5 or segment$5 or block$5) | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 15:13 |
| S31 | 50 | S29 and S30 and (mathmat$6 or algebr$5 or formul$5 or unity or discreet) | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 15:15 |
| S32 | 3210433 | image or picture or video or movie | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 15:12 |
| S33 | 1919 | encrypt$5.ab. and decrypt$5.ab. and (S32 or television or T).ab. | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 15:15 |
| S34 | 585958 | (movie or television or image or video or TV of film$5) with (partition$5 or divid$5 or separat$5 or segment$5 or block$5) | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 15:14 |
| S35 | 1914 | encrypt$5.ab. and decrypt$5.ab. and (S32 or television or TV or film$4).ab. | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 15:15 |
| S36 | 383 | S34 and S35 | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 15:16 |
| S37 | 131 | S36 and (mathmat$6 or algebr$5 or formul$5 or unity or discreet or power or integral) | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 15:18 |
| S38 | 147 | S36 and (mathmat$6 or algebr$5 or formul$5 or unity or discreet or power or integral or integer) | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 15:18 |
| S39 | 85 | S38 and @ad<"20010330" | US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB | OR | ON | 2005/06/11 15:18 |
| S40 | 10 | ("6195368").URPN. | USPAT | OR | ON | 2005/06/11 15:31 |

| | | | | | | |
|---|---|---|---|---|---|---|
| S41 | 1 | ("6192127").URPN. | USPAT | OR | ON | 2005/06/11 15:36 |

root of unity

>> Jump to:
Wikipedia
Best of Web
Mentioned In

Search Google
Images
News
Blogs
Products

Close

Wikipedia
root of unity

In mathematics, the ***n*-th roots of unity** or **de Moivre numbers**, named after Abraham de Moivre (1667 - 1754), are complex numbers located on the unit circle. They form the vertices of a *n*-sided regular polygon with one vertex on 1.

**Definition**

For a given *n* the complex numbers *z* which solve

$$z^n = 1 \qquad (n \in \mathbb{N}_0)$$

are called the ***n*-th roots of unity**. There are *n* different *n*-th roots of unity.

The *n*-th roots of unity form a cyclic group of order *n* under multiplication with 1 as the identity element. A generator for this cyclic group is called **primitive *n*-th root of unity**.

**Examples**

The third roots of unity are

$$\left\{1, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}\right\}.$$

The primitive third roots of unity are

$$\left\{\frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}\right\}.$$

The fourth roots of unity are

$$\{1, +i, -1, -i\}.$$

The primitive fourth roots of unity are

$$\{+i, -i\}.$$

**Properties**

As a consequence of Euler's identity the $n$-th roots of unity can be written as

$$e^{2\pi ik/n} \qquad (k, n \in \mathbb{N}_0 \text{ and } k < n).$$

As long as $n$ is at least 2, these numbers add up to 0, a simple fact that is of constant use in mathematics. It can be proved in any number of ways, for example by recognising the sum as coming from a geometric progression.

The primitive $n$-th roots of unity are precisely the numbers of the form $\exp(2\pi i\ k/n)$ where $k$ and $n$ are coprime. Therefore, there are $\varphi(n)$ different primitive $n$-th roots of unity, where $\varphi(n)$ denotes Euler's phi function. These different roots of unity can be arranged to form the elements of a unitary matrix, and are thus orthogonal to each other. A detailed exposition of the orthogonality relationship is given in the article character group.

**Cyclotomic polynomials**

The $n$-th roots of unity are precisely the zeros of the polynomial $p(X) = X^n - 1$; the primitive $n$th roots of unity are precisely the zeros of the $n$th cyclotomic polynomial

$$\Phi_n(X) = \prod_{k=1}^{\phi(n)} (X - z_k)$$

where $z_1,\ldots,z_{\varphi(n)}$ are the primitive $n$-th roots of unity. The polynomial $\Phi_n(X)$ has integer coefficients and is irreducible over the rationals (i.e., cannot be written as a product of two positive-degree polynomials with rational coefficients). The case of prime $n$, which is easier than the general assertion, follows from Eisenstein's criterion.

Every $n$th root of unity is a primitive $d$th root of unity for exactly one positive divisor $d$ of $n$. This implies that

$$X^n - 1 = \prod_{d\,|\,n} \Phi_d(X).$$

This formula represents the factorization of the polynomial $X^n - 1$ into irreducible factors and can also be used to compute the cyclotomic polynomials recursively. The first few are

$$\Phi_1(X) = X - 1$$
$$\Phi_2(X) = X + 1$$
$$\Phi_3(X) = X^2 + X + 1$$
$$\Phi_4(X) = X^2 + 1$$
$$\Phi_5(X) = X^4 + X^3 + X^2 + X + 1$$
$$\Phi_6(X) = X^2 - X + 1$$

In general, if $p$ is a prime number, then all $p$th roots of unity except 1 are primitive $p$th roots, and we have

$$\Phi_p(X) = \frac{X^p - 1}{X - 1} = \sum_{k=0}^{p-1} X^k$$

Note that, contrary to first appearances, *not* all coefficients of all cyclotomic polynomials are 1, −1, or 0; the first polynomial where this occurs is $\Phi_{105}$, since $105=3\times5\times7$ is the first product of three odd primes.

### Cyclotomic fields

By adjoining a primitive $n$th root of unity to **Q**, one obtains the ***n*th cyclotomic field** $F_n$. This field contains all $n$th roots of unity and is the splitting field of the $n$th cyclotomic polynomial over **Q**. The field extension $F_n/\mathbf{Q}$ has degree $\varphi(n)$ and its Galois group is naturally isomorphic to the multiplicative group of units of the ring $\mathbf{Z}/n\mathbf{Z}$.

As the Galois group of $F_n/\mathbf{Q}$ is abelian, this is an abelian extension. Every subfield of a cyclotomic field is an abelian extension of the rationals. In these cases Galois theory can be written out quite explicitly

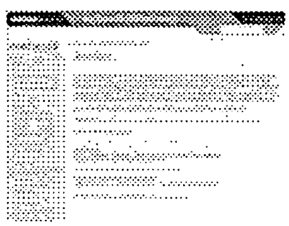
in terms of Gaussian periods: this theory from the *Disquisitiones Arithmeticae* of Gauss was published many years before Galois.

Conversely, *every* abelian extension of the rationals is such a subfield of a cyclotomic field - a theorem of Kronecker, usually called the *Kronecker-Weber theorem* on the grounds that Weber supplied the proof.



Best of the Web

Some good **"root of unity"** pages on the web:



Math
mathworld.wolfram.com

Mentioned In

*root unity* is mentioned in the following topics:

| | |
|---|---|
| Chowla-Mordell theorem | primitive root |
| trigonometric interpolation | Weil pairing |
| abelian extension | conjugate element (field theory) |
| Kronecker-Weber theorem | List of algebraic number theory topics |
| unit (ring theory) | Iwasawa theory |
| | More> |